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Diffraction of surface waves by floating elastic plates

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Abstract

Based on the dynamical theories of water waves and Mindlin thick plates, the diffraction of surface waves by a floating elastic plate is presented by using the Wiener–Hopf technique. Firstly, the problem is related to a wave guide in water of finite depth, which is analysed to determine the poles. The resulting hybrid boundary value problem is reduced to solving an infinite system of linear algebraic equations. The results obtained are compared with those calculated by an alternative analysis, and with experimental data. Finally, the effects of the geometric and physical parameters on the distribution of deflection and bending moments in plates are analysed and discussed.

Keywords: Diffraction of surface waves; Floating plate; Mindlin thick plate theory; Wiener–Hopf technique

1. Introduction

In recent years, the problem of the hydroelastic behaviour of a plate floating on the sea surface has received a great deal of attention. This interest was connected with the design of various floating platforms, artificial islands, airports, space-vehicle launch sites, etc. The dimensions of these structures are very large, which makes it difficult to satisfy the similarity criteria in carrying out experimental investigations. Therefore, numerical simulation is important in studying the hydroelastic behaviour of floating plates and the diffraction of surface waves.

Based on the eigenfunction expansion-matching method, Wu et al. (1995) analysed the wave-induced responses of an elastic floating plate by using modal expansions of the structural motion. Khabakhpasheva and Korobkin (2002) analysed simultaneously the plane problem of the hydroelastic behaviour of floating plates under the influence of periodic surface water waves based on hydroelasticity theory, and the expansions of the hydrodynamic pressure and the deflection made with respect to different basis functions. Linton and Chung (2003) solved the scattering of water waves by the edge of a semi-infinite ice sheet in a finite depth ocean by using the residue calculus technique. Andrianov and Hermans (2003, 2005) investigated the hydroelastic responses of a two-dimensional very large floating platform and a floating elastic circular plate to plane incident wave for three different cases, i.e., infinite, finite and shallow water depth.

Ohkusu and Namba (2004) presented an analytical approach to predict the bending vibration of a very large floating structure of thin and elongated rectangular plate configuration, floating on water of shallow depth and under the action of a monochromatic head wave. Fox and Squire (1990) computed the reflection and transmission coefficients by exactly solving the mathematical model. They used the complete set of modes to express the solutions with the coefficients

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found by matching through the water column beneath the edge of the ice sheet. This was performed numerically and led to a large system of equations that became unwieldy at short periods or large depths.

Diffraction of water waves by an elastic floating plate is a hybrid boundary value problem, in which one of the boundary conditions involves is a high order partial derivative. The Weiner–Hopf technique has many merits in solving such hybrid boundary value problems. With the aid of the Wiener–Hopf technique, some analytical solutions of the diffraction of surface waves by an elastic floating plate can be readily constructed, by extending the solution to the entire complex plane. More than 30 years ago, Evans and Davies (1968) formally solved such a mathematical model of floating sea ice using the Wiener–Hopf method, which gives the Fourier transform of the solution in each half plane, i.e. over the region of open water and the ice-covered region. Until now, the solution given by Evans and Davies has been thought to be unsuitable for actual computation because the required inverse Fourier transform is too difficult to obtain. Indeed, Evans and Davies had stated this opinion in their article.

Using the method of matched asymptotic expansions, Korobkin (2000) presented the solution of the plane unsteady problem of the hydroelastic behaviour of a plate floating on a liquid surface within the framework of linear theory. Balmforth and Craster (1999) studied the reflection and transmission of surface gravity waves incident on ice-covered ocean analytically. The ice cover is idealized as a plate of elastic material and the flexural motions were described by the Timoshenko–Mindlin equation. Tkacheva (2001, 2003) studied the hydroelastic behaviour of a floating semi-infinite plate for surface waves of finite water depth by using the Wiener–Hopf technique and presented a new approach to determine two unknown constants.

So far, in dealing with the diffraction of surface waves by floating plates the equations describing the flexural motions of the plate have usually been based on the dynamic theory of thin plates or Euler beam. However, the classical theory of thin plates in the analysis of dynamic problems has considerable limitations. Inclusion of the effects of transverse shearing deformation and rotary inertia in the dynamics of Mindlin thick plates leads to improved results for practical cases. In this paper, the investigation of the wave-induced responses of plane incident waves on an elastic floating plate is presented on the basis of the dynamical theory of Mindlin thick plates or Timoshenko beams, and the Weiner–Hopf technique.

2. Governing equations and the solutions

2.1. Differential equations of the fluid motion

The fluid is supposed to be incompressible and inviscid and the flow is assumed to be irrotational. We investigate the hydroelastic behaviour of a floating plate in waves on a fluid of finite depth a within the linear theory as shown in Fig. 1. The left edge of the plate is taken as the origin of the Cartesian coordinate system (x,z). The plate has a constant thickness h and length L. It is supposed that the plate thickness is significantly smaller than the length of the incident waves. The surface waves are concentrated in a thin layer on the fluid surface and decay exponentially with depth. The layer thickness is of the order of the wavelength. If the wavelength is comparable with the plate thickness, it is necessary to take into account the draft of the plate and wave reflection from its end; in this case almost all the energy of the surface waves will be reflected. The surface waves can penetrate into the plate if their length is significantly greater than the plate thickness. Therefore, we will neglect the draft of the platform and displace the boundary conditions to the unperturbed water surface. We assume that incoming waves are propagating along the positive x-direction.

We here consider the one-dimensional form of the dynamical theory of Mindlin thick plates (Hu, 1981), i.e. Timoshenko beam theory, in which all mechanical quantities in the plate depend only on the coordinate x. We can therefore obtain the following expressions:

$$\psi = \frac{\partial F}{\partial x}, \quad M = -D\frac{\partial^2 F}{\partial x^2}, \quad Q = C\left(\frac{\partial w}{\partial x} - \psi\right), \tag{1a,b,c}$$

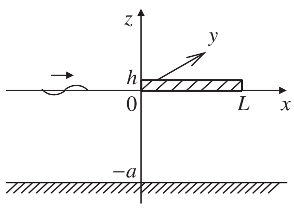

Fig. 1. Schematic of the fluid–structure interaction problem.

where ψ is the angle of rotation, M is the bending moment per unit width, Q is the shear force per unit width, $D = Eh^3/12(1-v^2)$ is the modulus of rigidity of the plate per unit width, E and v are Young's modulus and Poisson's ratio, respectively, $C = \varepsilon Gh$ is the shearing rigidity, $\varepsilon = \pi^2/12$ is the shear reduced factor, G is the shear modulus of elasticity, w is the vertical displacement of the upper surface of the fluid (i.e., the displacement of the plate), and F is a generalized function which satisfies the following differential equation:

$$w = \left(1 + \frac{\rho_0 J}{C}\frac{\partial^2}{\partial t^2} - \frac{D}{C}\frac{\partial^2}{\partial x^2}\right)F, \tag{2}$$

where J is the rotational inertia, ρ_0 is the plate density, and t is the time.

The solution of Eq. (2) can be expressed in the following form by Green's function:

$$F = \frac{C}{D}\int_0^L G(x,x')w(x')\,\mathrm{d}x', \tag{3a}$$

which $G(x,x')$ is Green's function of Eq. (2). Its finite representation (Hu, 1989) is

$$G(x,x') = \begin{cases} \dfrac{\sinh \gamma x}{\gamma \sinh \gamma L}\sinh \gamma(L-x'), & 0 \leqslant x < x', \\ \dfrac{\sinh \gamma(L-x)}{\gamma \sinh \gamma L}\sinh \gamma x', & x' \leqslant x < L, \end{cases} \tag{3b}$$

where $\gamma = \sqrt{(C - \rho_0 J\omega^2)/D}$ is the wavenumber of shearing vibration of the plate and ω is the circular frequency of the plate's transverse oscillation, equal to the frequency of the water waves. Therefore, the wave equation of the plate in the domain $z = 0, 0 < x < L$ is written as

$$D\frac{\partial^4 w}{\partial x^4} - \rho_0 J\left(1 + \frac{Dh}{JC}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \rho_0 h\frac{\partial^2 w}{\partial t^2} + \frac{\rho_0^2 Jh}{C}\frac{\partial^4 w}{\partial t^4} = \left(1 + \frac{\rho_0 J}{C}\frac{\partial^2}{\partial t^2} - \frac{D}{C}\frac{\partial^2}{\partial x^2}\right)p, \tag{4a}$$

where p is the hydrodynamic pressure, g is the acceleration due to gravity and ρ is the fluid density. The quantities w, F, p, φ contain the time factor $\mathrm{e}^{-\mathrm{i}\omega t}$.

The fluid velocity potential φ in the fluid region must satisfy Laplace's equation

$$\nabla^2 \varphi = 0 \quad (-a < z < 0). \tag{4b}$$

The boundary conditions on the free surface, the interface between the plate and fluid and the fluid bottom are, respectively, given by

$$p = -\rho\left(\frac{\partial \varphi}{\partial t} + gw\right) \quad (z = 0, \quad 0 < x < L), \tag{5a}$$

$$\rho\frac{\partial \varphi}{\partial t} + \rho g w = 0 \quad [z = 0, \quad x \in (-\infty, 0)U(L, \infty)], \tag{5b}$$

$$\frac{\partial \varphi}{\partial z} = \frac{\partial w}{\partial t} \quad (z = 0, \quad 0 < x < L), \tag{5c}$$

$$\frac{\partial \varphi}{\partial z} = 0 \quad (z = -a). \tag{5d}$$

Moreover, the bending moments and shearing forces at the plate edge must be equal to zero

$$\frac{\mathrm{d}^2 F}{\mathrm{d}x^2} = 0 \quad (x = 0, L), \tag{6a}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[w(x) - F(x)] = 0 \quad (x = 0, L). \tag{6b}$$

The following dimensionless variables are introduced:

$$\tilde{\varphi} = \frac{\varphi}{A\sqrt{gl}}, \quad \tilde{x} = x/a, \quad \tilde{z} = z/a, \quad \tilde{p} = \frac{p}{\rho g A}, \quad \tilde{k} = ka, \quad \tilde{L} = L/a, \quad \tilde{l} = l/a, \quad \tilde{h} = h/a, \quad \tilde{\rho} = \rho/\rho_0,$$

where A is the incident wave amplitude, $l = g/\omega^2$ and a is the fluid depth which is considered as the characteristic length. In what follows, we will employ these variables in the dimensionless forms and omit the tilde for convenience of writing.

The total velocity potential φ in the flow field can be expressed in the following form:

$$\varphi = (\varphi^{(i)} + \varphi^{(s)})\mathrm{e}^{-\mathrm{i}t}, \tag{7}$$

where $\varphi^{(i)}$ is the incident wave potential and $\varphi^{(s)}$ is the scattering potential. The waves and the vibration of the structure are of small amplitude and the water depth is constant. Therefore, the dimensionless potential $\varphi^{(i)}$ of the incident wave is

$$\varphi^{(i)} = \frac{\mathrm{e}^{\mathrm{i}kx}\cosh k(z+1)}{\cosh k},$$

where k is the incident wavenumber.

The scattered wave potential $\varphi^{(s)}$ in the region, $z = 0, 0 < x < L$, satisfies the following boundary condition:

$$H(x) = B(k)\mathrm{e}^{\mathrm{i}kx}, \tag{8a}$$

where

$$B(\alpha) = -[\beta(\alpha) + lb(\alpha)]\alpha\tanh(\alpha) + b(\alpha),$$

$$b(\alpha) = \frac{\rho\kappa^4}{h}\left[1 - \frac{\kappa^4 h^4}{6\pi^2(1-\nu)} + \frac{2h^2\alpha^2}{\pi^2(1-\nu)}\right],$$

$$\beta(\alpha) = \alpha^4 - \kappa^4 h^2\left[\frac{2}{\pi^2(1-\nu)} + \frac{1}{12}\right]\alpha^2 - \left[1 - \frac{h^4\kappa^4}{6\pi^2(1-\nu)}\right]\kappa^4,$$

$$H(x) = \left\{\frac{\partial^4}{\partial x^4} + \kappa^4 h^2\left[\frac{1}{12} + \frac{2}{\pi^2(1-\nu)}\right]\frac{\partial^2}{\partial x^2} - \kappa^4\left[1 - \kappa^4\frac{h^4}{6\pi^2(1-\nu)}\right]\right.$$
$$\left. + \frac{\rho l\kappa^4}{h}\left[1 - \frac{h^4\kappa^4}{6\pi^2(1-\nu)} - \frac{2h^2}{\pi^2(1-\nu)}\frac{\partial^2}{\partial x^2}\right]\right\}\frac{\partial\varphi^{(s)}}{\partial z} - \kappa^4\frac{\rho}{h}\left[1 - \frac{h^4\kappa^4}{6\pi^2(1-\nu)} - \frac{2h^2}{\pi^2(1-\nu)}\frac{\partial^2}{\partial x^2}\right]\varphi^{(s)},$$

and where $\kappa_0 = (\rho h\omega^2/D)^{1/4}$ is the wavenumber of flexural waves in the plate based on the classical theory of thin plates and $\kappa = \kappa_0 a$.

Then, from boundary conditions (5b) and (5c), the scattered potential $\varphi^{(s)}$ in the region $z = 0,\ \ x \in (-\infty, 0) \cup (L, \infty)$ satisfies the following boundary condition:

$$l\frac{\partial\varphi^{(s)}}{\partial z} - \varphi^{(s)} = 0. \tag{8b}$$

2.2. The dispersion relations

The wave propagation in the flow field is studied to determine the guided wave modes. In the free surface region away from the floating elastic plate, the dispersion relation is

$$K_1(\alpha) = \alpha l\tanh\alpha - 1 = 0, \tag{9}$$

where α is the wavenumber. Eq. (9) has two real roots $\pm k$ and a denumerable set of purely imaginary roots $\pm k_n$ ($n = 1,2,\ldots,\infty$). The influences of the roots on wave propagation become stronger as the distance between the purely imaginary roots and the origin in the complex plane reduces. These purely imaginary roots satisfy the condition $|k_{n+1}| > |k_n|$ and they are located symmetrically about the real axis in the complex plane.

In the region covered by the floating plate, the dispersion relation is

$$K_2(\alpha) = [\beta(\alpha) + lb(\alpha)]\alpha\tanh\alpha - b(\alpha) = 0, \tag{10}$$

which has two real roots $\pm\alpha_0$, a denumerable set of purely imaginary roots $\pm\alpha_n$ ($n = 1,2,\ldots,\infty$) and four complex roots located symmetrically about the real and imaginary axis, namely, $\alpha_{-1} = -\bar{\alpha}_{-2} = -\alpha_{-3} = \bar{\alpha}_{-4}$. $K_2(\alpha)$ here is equivalent to $B(\alpha)$ defined after Eq. (8a). Fox and Squire (1990) have shown that these purely imaginary roots satisfy the condition $|\alpha_{n+1}| > |\alpha_n|$ and are located symmetrically about the real axis in the complex plane. We will denote the complex root in the ith quadrants as α_{-i} ($i = 1,2,3,4$), as shown in Fig. 2.

It is well known that the real roots of the dispersion relations denote propagating waves, the purely imaginary roots correspond to local excitations evanescent modes and the four complex roots denote decaying waves. It can be shown that the dispersion relations $K_1(\alpha)$ and $K_2(\alpha)$ are even.

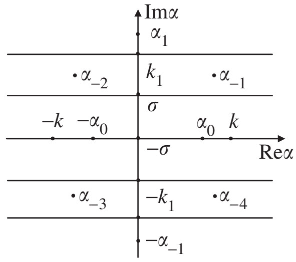

Fig. 2. The positions of the roots $K_1(\alpha) = 0$ and $K_2(\alpha) = 0$ in the complex plane.

2.3. The Wiener–Hopf technique

The problem is solved using the Wiener–Hopf technique (Noble, 1958; Tkacheva, 2001, 2003). The following functions of the complex variable α are introduced to convert the problem in the spatial domain into a problem in the wavenumber domain:

$$\Phi_+(\alpha, z) = \int_L^\infty e^{i\alpha(x-L)}\varphi^{(s)}(x, z)\,dx, \tag{11a}$$

$$\Phi_-(\alpha, z) = \int_{-\infty}^0 e^{i\alpha x}\varphi^{(s)}(x, z)\,dx, \tag{11b}$$

$$\Phi_1(\alpha, z) = \int_0^L e^{i\alpha x}\varphi^{(s)}(x, z)\,dx, \tag{11c}$$

$$\Phi(\alpha, z) = \Phi_-(\alpha, z) + \Phi_1(\alpha, z) + e^{i\alpha L}\Phi_+(\alpha, z). \tag{11d}$$

It is observed that $\Phi_+(\alpha,z)$ and $\Phi_-(\alpha,z)$ are defined in the half planes $\mathrm{Im}\,\alpha>0$ and $\mathrm{Im}\,\alpha<0$, respectively. With the principle of analytic continuation, these functions can be defined over the entire complex plane.

Now we will study the characteristics of the functions $\Phi_\pm(\alpha,z)$. As $x\to-\infty$, the scattered potential $\varphi^{(s)}$ is a reflected wave of the form Re^{-ikx}, a set of local excitations which do not propagate and a set of decaying waves. The least order local excitation mode corresponds to the root k_1. Therefore, $\Phi_-(\alpha,z)$ is analytic in the half-plane $\mathrm{Im}\,\alpha<|k_1|$ except for the pole at $\alpha = k$. As $x\to\infty$, the scattered potential $\varphi^{(s)}$ is a transmitted wave of the form Te^{-ikx}, a set of local excitations and a set of decaying waves. Therefore, the function $\Phi_+(\alpha,z)$ is analytical in the half-plane $\mathrm{Im}\,\alpha>-|k_1|$ except for the pole at $\alpha = -k$.

The function $\Phi(\alpha,z)$ is the Fourier transform of the function $\varphi(x,z)$ with respect to the space variable x and it must satisfy the equation $\partial\Phi/\partial z^2-\alpha^2\Phi = 0$. The general solution of the equation with the boundary condition (5d) on the seabed is given by

$$\Phi(\alpha, z) = Y(\alpha)\frac{\cosh[\alpha(z+1)]}{\cosh(\alpha)}. \tag{12}$$

We will apply the Fourier transform to the left-hand side of condition (8a), which can be denoted by $J_\pm(\alpha)$ and $J_1(\alpha)$ as follows:

$$J(\alpha) = J_-(\alpha) + J_1(\alpha) + e^{i\alpha L}J_+(\alpha), \tag{13}$$

where

$$J_-(\alpha) = \int_{-\infty}^0 H(x)e^{i\alpha x}\,dx, \quad J_+(\alpha) = \int_L^\infty H(x)e^{i\alpha(x-L)}\,dx,$$
$$J_1(\alpha) = \int_0^L H(x)e^{i\alpha x}\,dx = \int_0^L B(k)e^{i\alpha x}e^{ikx}\,dx = \frac{B(k)[e^{i(\alpha+k)L}-1]}{i(\alpha+k)}.$$

Similarly, applying Fourier transform to the left-hand side of condition (8b), we will denote these integrands by $X_\pm(\alpha)$ and $X_1(\alpha)$ as follows:

$$X(\alpha) = X_-(\alpha) + X_1(\alpha) + e^{i\alpha L}X_+(\alpha), \tag{14}$$

where

$$X_-(\alpha)=\int_{-\infty}^{0}\left(l\frac{\partial\varphi^{(s)}}{\partial z}-\varphi^{(s)}\right)\mathrm{e}^{\mathrm{i}\alpha x}\,\mathrm{d}x,\quad X_1(\alpha)=\int_{0}^{L}\left(l\frac{\partial\varphi^{(s)}}{\partial z}-\varphi^{(s)}\right)\mathrm{e}^{\mathrm{i}\alpha x}\,\mathrm{d}x,$$
$$X_+(\alpha)=\int_{L}^{\infty}\left(l\frac{\partial\varphi^{(s)}}{\partial z}-\varphi^{(s)}\right)\mathrm{e}^{\mathrm{i}\alpha(x-L)}\,\mathrm{d}x.$$

From the boundary condition (8b), we have the relation $X_-(\alpha)=X_+(\alpha)=0$ and obtain

$$X(\alpha)=X_1(\alpha). \tag{15}$$

According to Eq. (13), we have

$$J(\alpha)=\int_{-\infty}^{\infty}H(x)\mathrm{e}^{\mathrm{i}\alpha x}\,\mathrm{d}x. \tag{16}$$

After some algebra, the functions are found to satisfy the following relation:

$$J(\alpha)=[\beta(\alpha)+lb(\alpha)]\alpha Y(\alpha)\tanh(\alpha)-b(\alpha)Y(\alpha)=Y(\alpha)K_2(\alpha). \tag{17}$$

Similarly, we have

$$X(\alpha)=X_1(\alpha)=Y(\alpha)K_1(\alpha). \tag{18}$$

From Eqs. (13) and (17), we obtain

$$J_-(\alpha)+J_1(\alpha)+\mathrm{e}^{\mathrm{i}\alpha L}J_+(\alpha)=Y(\alpha)K_2(\alpha). \tag{19}$$

Substituting Eq. (18) into Eq. (19) and eliminating $Y(\alpha)$, we obtain the following equation:

$$J_-(\alpha)+\frac{B(k)[\mathrm{e}^{\mathrm{i}(\alpha+k)L}-1]}{\mathrm{i}(\alpha+k)}+\mathrm{e}^{\mathrm{i}\alpha L}J_+(\alpha)=X_1(\alpha)K(\alpha), \tag{20}$$

where $K(\alpha)=K_2(\alpha)/K_1(\alpha)$.

We factorize the function $K(\alpha)$ in the strip $-|k_1|<\operatorname{Im}\alpha<|k_1|$

$$K(\alpha)=K_+(\alpha)K_-(\alpha), \tag{21}$$

where the functions $K_\pm(\alpha)$ and $\Phi_\pm(\alpha,z)$ are regular in the same domain. The points $\pm k$ and $\pm\alpha_0$ are the zeros and poles, respectively, on the real axis for the function $K(\alpha)$. Therefore, it can be shown that S_+ is the upper half-plane $\operatorname{Im}\alpha>-|k_1|$ without the points $-\alpha_0$ and $-k$ and that S_- is the lower half-plane $\operatorname{Im}\alpha<|k_1|$ without the points α_0 and k.

We now introduce the function

$$g(\alpha)=\frac{K(\alpha)(\alpha^2-k^2)}{(\alpha^2-\alpha_0^2)(\alpha^2-\alpha_{-1}^2)(\alpha^2-\alpha_{-2}^2)}. \tag{22}$$

It is observed that $g(\alpha)$ is analytic in the strip $-|k_1|<\operatorname{Im}\alpha<|k_1|$ after eliminating the zeros and poles of the function $K(\alpha)$. Moreover, it is obvious that on the real axis the function $g(\alpha)$ has no zeros, has boundaries, and tends to unity at infinity. The function $g(\alpha)$ can be factorized as follows:

$$g(\alpha)=g_+(\alpha)g_-(\alpha),\quad g_\pm(\alpha)=\exp\left[\pm\frac{1}{2\pi\mathrm{i}}\int_{-\infty\mp\mathrm{i}\sigma}^{\infty\mp\mathrm{i}\sigma}\frac{\ln g(x)}{x-\alpha}\,\mathrm{d}x\right]\quad(\sigma<|k_1|). \tag{23}$$

Following the theorem of Noble (1958), the function $g(\alpha)$ can also be expressed as an infinite product

$$g_\pm(\alpha)=\sqrt{g(0)}\prod_{n=1}^{\infty}\left[\left(1\pm\frac{\alpha}{\alpha_n}\right)\exp(\mp\mathrm{i}\alpha/\alpha_n\pm\mathrm{i}\alpha/k_n)\Big/\left(1\pm\frac{\alpha}{k_n}\right)\right]. \tag{24}$$

We define the functions $K_\pm(\alpha)$ as follows:

$$K_\pm(\alpha)=\frac{(\alpha\pm\alpha_0)(\alpha\pm\alpha_{-1})(\alpha\pm\alpha_{-2})}{\alpha\pm k}g_\pm(\alpha). \tag{25}$$

It can be shown that from the above expressions the functions $K_\pm(\alpha)$ satisfy the relation $K_\pm(\alpha)=K_-(-\alpha)$. We multiply both sides of Eq. (20) by $\mathrm{e}^{-\mathrm{i}\alpha L}[K_\pm(\alpha)]^{-1}$ and rearrange it in the following form:

$$\frac{J_-(\alpha)\mathrm{e}^{-\mathrm{i}\alpha L}}{K_+(\alpha)}+\frac{B(k)(\mathrm{e}^{\mathrm{i}kL}-\mathrm{e}^{-\mathrm{i}\alpha L})}{\mathrm{i}(\alpha+k)K_+(\alpha)}+\frac{J_+(\alpha)}{K_+(\alpha)}=X_1(\alpha)K_-(\alpha)\mathrm{e}^{-\mathrm{i}\alpha L} \tag{26a}$$

or

$$\frac{J_+(\alpha)}{K_+(\alpha)}+\frac{B(k)\mathrm{e}^{\mathrm{i}kL}}{\mathrm{i}(\alpha+k)K_+(\alpha)}+U_+(\alpha)-V_+(\alpha)=X_1(\alpha)K_-(\alpha)\mathrm{e}^{-\mathrm{i}\alpha L}-U_-(\alpha)+V_-(\alpha), \tag{26b}$$

where

$$U_\pm(\alpha)=\pm\frac{1}{2\pi\mathrm{i}}\int_{-\infty\mp\mathrm{i}\sigma}^{\infty\mp\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}J_-(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)},\quad V_\pm(\alpha)=\mp\frac{B(k)}{2\pi}\int_{-\infty\mp\mathrm{i}\sigma}^{\infty\mp\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta+k)(\zeta-\alpha)}\quad(\sigma<\sigma_0),$$

with $\sigma_0=\min(|k_1|,|\alpha_{-1}|)$.

The left-hand side of Eq. (26b) is analytic in region S_+, while the right-hand side functions are analytic in region S_-. Applying the principle of analytic continuation, we define the function over the entire complex plane. According to Liouville's theorem, the left-hand side of Eq. (26b) must be a polynomial, in which the degree of the polynomial can be determined by the behaviour of the functions as $|\alpha|\to\infty$. From Eqs. (12) and (13), we can see that as $|\alpha|\to\infty$ the function $J_-(\alpha)$ is of order no higher than $\mathcal{O}(|\alpha|^{\lambda+3})(\lambda<1)$ and the function $X_+(\alpha)$ is of order no higher than $\mathcal{O}(|\alpha|^{\lambda-1})$. As $|\alpha|\to\infty$, $g_\pm(\alpha)$ tend to unity and the functions $K_\pm(\alpha)$ are of the order $\mathcal{O}(|\alpha|^2)$. Thus, we obtain the following equation:

$$\frac{J_+(\alpha)}{K_+(\alpha)}+\frac{B(k)\mathrm{e}^{\mathrm{i}kL}}{\mathrm{i}(\alpha+k)K_+(\alpha)}+U_+(\alpha)-V_+(\alpha)=a_1\alpha+b_1. \tag{27}$$

Multiplying both sides of Eq. (20) by $[K_-(\alpha)]^{-1}$, we obtain the following equation:

$$\frac{J_-(\alpha)}{K_-(\alpha)}+R_-(\alpha)-S_-(\alpha)-\frac{B}{\mathrm{i}(\alpha+k)}\left[\frac{1}{K_-(\alpha)}-\frac{1}{K_-(-k)}\right]=X_1(\alpha)K_+(\alpha)-R_+(\alpha)+S_+(\alpha)+\frac{B(k)}{\mathrm{i}(\alpha+k)K_+(k)}, \tag{28}$$

where

$$R_+(\alpha)+R_-(\alpha)=\frac{\mathrm{e}^{\mathrm{i}\alpha L}J_+(\alpha)}{K_-(\alpha)},\quad R_\pm(\alpha)=\pm\tfrac{1}{2\pi\mathrm{i}}\int_{-\infty\mp\mathrm{i}\sigma}^{\infty\mp\mathrm{i}\sigma}\frac{\mathrm{e}^{\mathrm{i}\zeta L}J_+(\zeta)}{K_-(\zeta)(\zeta-\alpha)}\mathrm{d}\zeta\quad(\sigma<\sigma_0),$$
$$S_+(\alpha)+S_-(\alpha)=-\frac{B(k)\mathrm{e}^{\mathrm{i}(\alpha+k)L}}{\mathrm{i}(\alpha+k)K_-(\alpha)},\quad S_\pm(\alpha)=\pm\tfrac{B(k)}{2\pi}\int_{-\infty\mp\mathrm{i}\sigma}^{\infty\mp\mathrm{i}\sigma}\frac{\mathrm{e}^{\mathrm{i}(\zeta+k)L}J_+(\zeta)\,\mathrm{d}\zeta}{K_-(\zeta)(\zeta+k)(\zeta-\alpha)}\quad(\sigma<\sigma_0).$$

Similarly, from Eq. (28) we obtain

$$\frac{J_-(\alpha)}{K_-(\alpha)}+R_-(\alpha)-S_-(\alpha)-\frac{B(k)}{\mathrm{i}(\alpha+k)}\left[\frac{1}{K_-(\alpha)}-\frac{1}{K_-(-k)}\right]=a_2\alpha+b_2, \tag{29}$$

where a_1, b_1, a_2, b_2 are unknown constants.

In the spatial wavenumber domain, we introduce the new unknown functions

$$\Psi_+(\alpha)=J_+(\alpha)+\frac{B(k)\mathrm{e}^{\mathrm{i}kL}}{\mathrm{i}(\alpha+k)}, \tag{30a}$$

$$\Psi^*_-(\alpha)=J_-(\alpha)-\frac{B(k)}{\mathrm{i}(\alpha+k)}, \tag{30b}$$

where the superscript star is used to indicate that apart from the pole at $\alpha=-k$ the functions $\Psi^*_-(\alpha)$ and $J_-(\alpha)$ are all analytic in the domain S_-. Substituting Eq. (30) into Eqs. (27) and (29), we obtain

$$\frac{\Psi_+(\alpha)}{K_+(\alpha)}+\frac{1}{2\pi\mathrm{i}}\int_{-\infty-\mathrm{i}\sigma}^{\infty-\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi^*_-(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)}=a_1\alpha+b_1\quad(\sigma<\sigma_0), \tag{31a}$$

$$\frac{\Psi^*_-(\alpha)}{K_-(\alpha)}+\frac{B(k)}{\mathrm{i}(\alpha+k)K_-(-k)}-\frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma}\frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)}{K_-(\zeta)(\zeta-\alpha)}\,\mathrm{d}\zeta=a_2\alpha+b_2\quad(\sigma<\sigma_0). \tag{31b}$$

By means of the boundary conditions (6a) and (6b), we can obtain the expressions for the four unknown constants a_1, b_1, a_2 and b_2 (see Appendix A):

$$
\begin{aligned}
a_1 = {} & \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[\chi_s N_1(\chi_s)A_{22} - N_2(\chi_s)A_{12}]}{\beta'(\chi_s)K_-(\chi_s)(A_{11}A_{22} - A_{12}A_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)\,\mathrm{d}\alpha}{K_+(\alpha)(-\alpha + \chi_s)} \\
& + \frac{b(\mathrm{i}\gamma)(\mathrm{i}\gamma A_{22} - qA_{12})}{2\beta(\mathrm{i}\gamma)(A_{11}A_{22} - A_{12}A_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)}{K_+(\alpha)K_-(\mathrm{i}\gamma)(-\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha \\
& + \frac{b(\mathrm{i}\gamma)(\mathrm{i}\gamma A_{22} + qA_{12})}{2\beta(\mathrm{i}\gamma)(A_{11}A_{22} - A_{12}A_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)}{K_+(\alpha)K_-(-\mathrm{i}\gamma)(\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha,
\end{aligned} \tag{32a}
$$

$$
\begin{aligned}
b_1 = {} & \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[\chi_s N_1(\chi_s)A_{21} - N_2(\chi_s)A_{11})]}{\beta'(\chi_s)K_-(\chi_s)(A_{12}A_{21} - A_{11}A_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)\,\mathrm{d}\alpha}{K_+(\alpha)(-\alpha + \chi_s)} \\
& + \frac{b(\mathrm{i}\gamma)(\mathrm{i}\gamma A_{21} - qA_{11})}{2\beta(\mathrm{i}\gamma)(A_{12}A_{21} - A_{11}A_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)}{K_+(\alpha)} \frac{1}{K_-(\mathrm{i}\gamma)(-\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha \\
& + \frac{b(\mathrm{i}\gamma)(\mathrm{i}\gamma A_{21} + qA_{11})}{2\beta(\mathrm{i}\gamma)(A_{12}A_{21} - A_{11}A_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi_-^*(\alpha)}{K_+(\alpha)} \frac{1}{K_-(-\mathrm{i}\gamma)(\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha.
\end{aligned} \tag{32b}
$$

$$
\begin{aligned}
a_2 = {} & \frac{P_{22}Q_1 - P_{12}Q_2}{(P_{11}P_{22} - P_{12}P_{21})} + \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[N_2(\chi_s)P_{12} - \chi_s N_1(\chi_s)P_{22}]}{\beta'(\chi_s)K_+(\chi_s)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)\,\mathrm{d}\alpha}{K_-(\alpha)(\alpha - \chi_s)} \\
& + \frac{b(\mathrm{i}\gamma)(qP_{12} - \mathrm{i}\gamma P_{22})}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(\mathrm{i}\gamma)(\alpha - \mathrm{i}\gamma)}\,\mathrm{d}\alpha \\
& + \frac{b(\mathrm{i}\gamma)(qP_{12} + \mathrm{i}\gamma P_{22})}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(-\mathrm{i}\gamma)(\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha,
\end{aligned} \tag{33a}
$$

$$
\begin{aligned}
b_2 = {} & \frac{P_{21}Q_1 - P_{11}Q_2}{(P_{12}P_{21} - P_{11}P_{22})} + \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[N_2(\chi_s)P_{11} - \chi_s N_1(\chi_s)P_{21}]}{\beta'(\chi_s)K_+(\chi_s)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)\,\mathrm{d}\alpha}{K_-(\alpha)(\alpha - \chi_s)} \\
& + \frac{b(\mathrm{i}\gamma)(qP_{11} - \mathrm{i}\gamma P_{21})}{2\beta(\mathrm{i}\gamma)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(\mathrm{i}\gamma)(\alpha - \mathrm{i}\gamma)}\,\mathrm{d}\alpha \\
& + \frac{b(\mathrm{i}\gamma)(qP_{11} + \mathrm{i}\gamma P_{21})}{2\beta(\mathrm{i}\gamma)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(-\mathrm{i}\gamma)(\alpha + \mathrm{i}\gamma)}\,\mathrm{d}\alpha.
\end{aligned} \tag{33b}
$$

Substituting Eqs. (32a), (32b), (33a) and (33b) for the coefficients a_1, b_1, a_2 and b_2 into Eqs. (31a) and (31b) and deforming the contours to have the same integration contour in each equation, we can obtain the following system of integral equations:

$$
\begin{aligned}
& \frac{\Psi_+(\alpha)}{K_+(\alpha)} + \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta - \alpha)} \\
& \quad - \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[\chi_s N_1(\chi_s)(A_{22}\alpha - A_{21}) - N_2(\chi_s)(A_{12}\alpha - A_{11})]}{\beta'(\chi_s)K_-(\chi_s)(A_{11}A_{22} - A_{12}A_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(-\zeta + \chi_s)} \\
& \quad - \frac{b(\mathrm{i}\gamma)[\mathrm{i}\gamma(A_{22}\alpha - A_{21}) - q(A_{12}\alpha - A_{11})]}{2\beta(\mathrm{i}\gamma)(A_{11}A_{22} - A_{12}A_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)}{K_+(\zeta)K_-(\mathrm{i}\gamma)(-\zeta + \mathrm{i}\gamma)}\,\mathrm{d}\zeta,
\end{aligned} \tag{34a}
$$

$$
\begin{aligned}
& \frac{\Psi_-^*(\alpha)}{K_-(\alpha)} - \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)\,\mathrm{d}\zeta}{K_-(\zeta)(\zeta - \alpha)} \\
& \quad - \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[N_2(\chi_s)(P_{12}\alpha - P_{11}) - \chi_s N_1(\chi_s)(P_{22}\alpha - P_{21})]}{\beta'(\chi_s)K_+(\chi_s)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)\,\mathrm{d}\zeta}{K_-(\zeta)(\zeta - \chi_s)}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{b(\mathrm{i}\gamma)[q(P_{12}\alpha-P_{11})-\mathrm{i}\gamma(P_{22}\alpha-P_{21})]}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22}-P_{12}P_{21})}\frac{1}{2\pi\mathrm{i}}\int_{C_+}\frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)}{K_-(\zeta)K_+(\mathrm{i}\gamma)(\zeta-\mathrm{i}\gamma)}\,\mathrm{d}\zeta\\
&-\frac{b(\mathrm{i}\gamma)[q(P_{12}\alpha-P_{11})+\mathrm{i}\gamma(P_{22}\alpha-P_{21})]}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22}-P_{12}P_{21})}\frac{1}{2\pi\mathrm{i}}\int_{C_+}\frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)}{K_-(\zeta)K_+(-\mathrm{i}\gamma)(\zeta+\mathrm{i}\gamma)}\,\mathrm{d}\zeta\\
&=\frac{Q_1(P_{22}\alpha-P_{21})+Q_2(P_{11}-P_{12}\alpha)}{P_{11}P_{22}-P_{12}P_{21}}-\frac{B(k)}{\mathrm{i}(\alpha+k)K_-(-k)}.
\end{aligned}
\tag{34b}
$$

In order to numerically solve the above system of integral equations, we introduce the following new unknown functions:

$$
\xi(\alpha)=\frac{\Psi_+(\alpha)}{K_+(\alpha)},\quad \eta(\alpha)=\frac{\Psi^*_-(\alpha)}{K_-(\alpha)}. \tag{35a,b}
$$

Substituting Eqs. (35a) and (35b) into the above system of equations, we will evaluate the integrals using the residue calculus technique. After substituting $\alpha=\alpha_j$ ($j=-2,-1,0,1,\ldots$) into the first equation and $\alpha=-\alpha_j$ ($j=-2,-1,0,1,\ldots$) into the second equation, we close the integration contour in the lower half-plane in the first equation and in the upper half-plane in the second equation. Applying the residue calculus technique, the system of integral equations is reduced to the infinite system of algebraic equations

$$
\begin{aligned}
\xi_j+\sum_{m=-2}^{\infty}\eta_m\frac{\mathrm{e}^{\mathrm{i}\alpha_m L}K_+^2(\alpha_m)K_1(\alpha_m)}{K_2'(-\alpha_m)}\Bigg\{&\frac{1}{\alpha_m+\alpha_j}+\sum_{s=1}^{4}\frac{\chi_s b(\chi_s)[\chi_s N_1(\chi_s)(A_{22}\alpha_j-A_{21})-N_2(\chi_s)(A_{12}\alpha_j-A_{11})]}{\beta'(\chi_s)K_-(\chi_s)(A_{11}A_{22}-A_{12}A_{21})(\alpha_m+\chi_s)}\\
&+\frac{b(\mathrm{i}\gamma)[\mathrm{i}\gamma(A_{22}\alpha_j-A_{21})-q(A_{12}\alpha_j-A_{11})]}{2\beta(\mathrm{i}\gamma)(A_{11}A_{22}-A_{12}A_{21})K_-(\mathrm{i}\gamma)(\alpha_m+\mathrm{i}\gamma)}+\frac{b(\mathrm{i}\gamma)[\mathrm{i}\gamma(A_{22}\alpha_j-A_{21})+q(A_{12}\alpha_j-A_{11})]}{2\beta(\mathrm{i}\gamma)(A_{11}A_{22}-A_{12}A_{21})K_-(-\mathrm{i}\gamma)(-\alpha_m+\mathrm{i}\gamma)}\Bigg\}=0,
\end{aligned}
\tag{36a}
$$

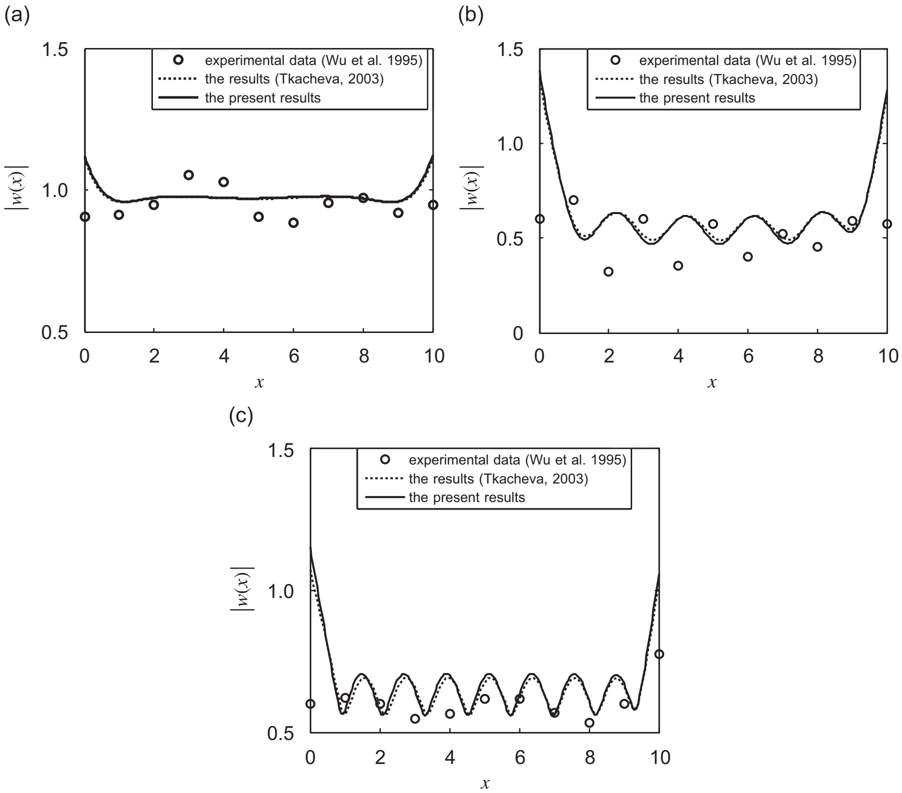

Fig. 3. Amplitude of the displacement over the plate for different dimensionless wavenumbers. (a) $k=0.8044$, (b) $k=2.2216$, and (c) $k=9.0434$.

$$\eta_j + \sum_{m=-2}^{\infty} \xi_m \frac{e^{i\alpha_m L} K_+^2(\alpha_m) K_1(\alpha_m)}{K_2'(\alpha_m)} \left\{ -\frac{1}{\alpha_m + \alpha_j} - \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[-N_2(\chi_s)(P_{12}\alpha_j + P_{11}) + \chi_s N_1(\chi_s)(P_{22}\alpha_j + P_{21})]}{\beta'(\chi_s) K_+(\chi_s)(P_{11}P_{22} - P_{12}P_{21})(\alpha_m - \chi_s)} \right.$$
$$\left. - \frac{b(i\gamma)[-q(P_{12}\alpha_j + P_{11}) + i\gamma(P_{22}\alpha_j + P_{21})]}{2\beta(i\gamma)(P_{11}P_{22} - P_{12}P_{21})K_+(i\gamma)(\alpha_m - i\gamma)} - \frac{b(i\gamma)[-q(P_{12}\alpha_j + P_{11}) - i\gamma(P_{22}\alpha_j + P_{21})]}{2\beta(i\gamma)(P_{11}P_{22} - P_{12}P_{21})K_+(-i\gamma)(\alpha_m + i\gamma)} \right\}$$
$$= \frac{-Q_1(P_{22}\alpha_j + P_{21}) + Q_2(P_{11} + P_{12}\alpha_j)}{P_{11}P_{22} - P_{12}P_{21}} - \frac{B(k)}{i(k - \alpha_j)K_-(-k)}, \tag{36b}$$

where $\xi_j = \xi(\alpha_j)$, $\eta_j = \eta(-\alpha_j)$. Therefore, the discrete values of the unknown functions $\xi_j = \xi(\alpha_j)$, $\eta_j = \eta(-\alpha_j)$ can be determined by solving Eqs. (36). It can be shown that the system (36) satisfies the reduction conditions (Kantorovich and Akilov, 1977) and the solution of the finite reduced system converges to the solution of the initial system when the order of the finite system tends to infinity.

3. Wave-induced response, reflection and transmission coefficients

We will calculate the deflection of the floating plate. According to Eqs. (13), (20), (30a) and (30b), we have

$$Y(\alpha) = \frac{1}{K_2(\alpha)}[\Psi_+(\alpha)e^{i\alpha L} + \Psi_-^*(\alpha)]. \tag{37}$$

Thus, from Eqs. (12), (37), we obtain the following expression for the scattered potential $\varphi^{(s)}$ by Fourier inversion:

$$\varphi^{(s)}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-i\alpha(x-L)}\cosh\alpha(z+1)\Psi_+(\alpha)\,d\alpha}{K_2(\alpha)\cosh(\alpha)} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{-i\alpha x}\cosh\alpha(z+1)\Psi_-^*(\alpha)\,d\alpha}{K_2(\alpha)\cosh(\alpha)}. \tag{38}$$

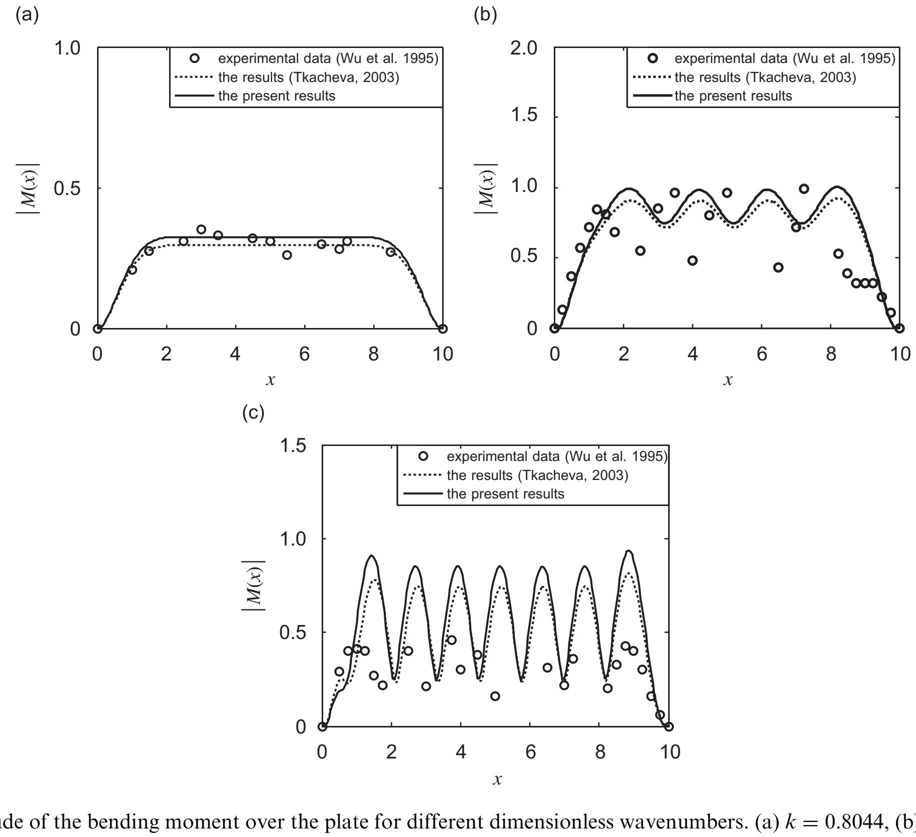

Fig. 4. Amplitude of the bending moment over the plate for different dimensionless wavenumbers. (a) $k = 0.8044$, (b) $k = 2.2216$, and (c) $k = 9.0434$.

After derivation of Eq. (38) with respect to $z=0$, we obtain

$$\frac{\partial}{\partial z}\varphi^{(s)}(x,0)=\mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)}{K_2'(\alpha_m)}[\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}\Psi_+(\alpha_m)+\mathrm{e}^{\mathrm{i}\alpha_m x}\Psi_-^*(-\alpha_m)]-\mathrm{e}^{\mathrm{i}kx}. \quad (39)$$

From the boundary condition (3c) and Eq. (39), we obtain the following expression for the plate deflection:

$$w(x)=-\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)K_+(\alpha_m)}{K_2'(\alpha_m)}[\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}\xi_m+\mathrm{e}^{\mathrm{i}\alpha_m x}\eta_m]. \quad (40)$$

It can be shown in Eq. (40) that the value of ξ_j determines the complex amplitudes of the elastic waves travelling from the right-hand edge of the plate and the value of η_j determines the amplitudes of elastic waves travelling from the left-hand edge.

According to Eq. (40) and (3a), we have

$$F(x)=\sum_{m=-2}^{\infty}\frac{q\alpha_m\tanh(\alpha_m)K_+(\alpha_m)N_1(\alpha_m)}{K_2'(\alpha_m)}[\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}\xi_m+\mathrm{e}^{\mathrm{i}\alpha_m x}\eta_m]. \quad (41)$$

From Eqs. (1b) and (41), the dimensionless bending moment $|M(x)|$ is given in the following form:

$$\left|M(x)\right|=\frac{D}{\rho g L_0\,\mathrm{d}a^2}\left|\sum_{m=-2}^{\infty}\frac{q\alpha_m^3\tanh(\alpha_m)K_+(\alpha_m)N_1(\alpha_m)}{K_2'(\alpha_m)}[\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}\xi_m+\mathrm{e}^{\mathrm{i}\alpha_m x}\eta_m]\right|. \quad (42)$$

Next, we will calculate the reflection and transmission coefficients. For the reflected wave, as x tends to $-\infty$, $\varphi^{(s)}(x{,}0)=R\mathrm{e}^{-\mathrm{i}kx}$. Thus the modulus $|R|$ of the coefficient of $\exp(-\mathrm{i}kx)$ is the reflection coefficient. We take the representation for the scattered potential $\varphi^{(s)}$ in the form of Eq. (A.22) and determine the value of R by evaluating the

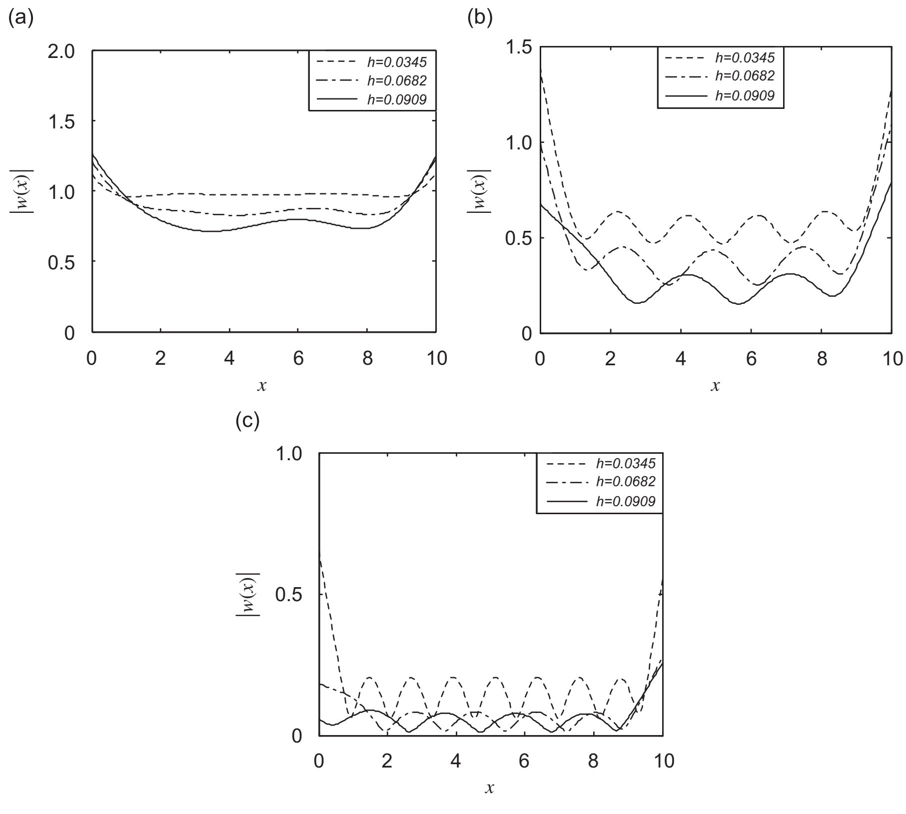

Fig. 5. Amplitude of the displacement over the plate for different dimensionless thicknesses. (a) $k=0.8044$, (b) $k=2.2216$, and (c) $k=9.0434$.

residue at the point $\alpha = k$, namely

$$R = \mathrm{i}\left\{\frac{1}{K_+(k)K_1'(k)}\left[a_2k + b_2 - \frac{B(k)}{2\mathrm{i}kK_+(k)} + \frac{1}{2\pi\mathrm{i}}\int_{-\infty-\mathrm{i}\sigma}^{\infty-\mathrm{i}\sigma}\frac{\mathrm{e}^{\mathrm{i}\zeta L}\Psi_+(\zeta)\,\mathrm{d}\zeta}{K_-(\zeta)(\zeta-k)}\right]\right\}. \tag{43}$$

For the transmitted wave, as x tends to ∞, $\varphi(x,0) = T\mathrm{e}^{\mathrm{i}kx}$. We take the representation for $\varphi^{(s)}$ in the form of Eq. (A.3) and determine the value of T by evaluating the residue in (A.3) at the point $\alpha = -k$, namely

$$T = 1 + \frac{\mathrm{i}e^{-\mathrm{i}kL}}{K_-(-k)K_1'(-k)}\left[b_1 - a_1k - \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(k)(\zeta+k)}\right]. \tag{44}$$

4. Numerical results

For the purpose of checking the validity of the present method, numerical calculations are performed for a physical model, which has been investigated experimentally by Wu et al. (1995). The parameters of the model are: Young's modulus $E = 103\,\mathrm{MPa}$, Poisson's ratio $v = 0.3$, the plate length $L = 10\,\mathrm{m}$, the plate thickness $h = 38\,\mathrm{mm}$, the draft $d = 8.36\,\mathrm{mm}$, the fluid density $\rho = 1000\,\mathrm{kg/m^3}$, the water depth $a = 1.1\,\mathrm{m}$, and the density ratio between the fluid and plate $\rho/\rho_0 = 4.5455$. The periods of the incident wave are equal to 2.875, 1.429 and 0.7 s, respectively. Corresponding to the incident-wave periods, the dimensionless wave numbers of the incident wave are $k = 0.8044$, 2.2216, 9.0434, respectively.

For the nondimensional plate thickness $h/a = 0.0345$, the calculated amplitudes of both the plate deflection $|w(x)|$ and bending moments $|M(x)|$ by using the present method are in good agreement with the results by Tkacheva (2003) obtained based on Weiner–Hopf technique and classical dynamics theory of thin plates, as shown in Figs. 3 and 4. It

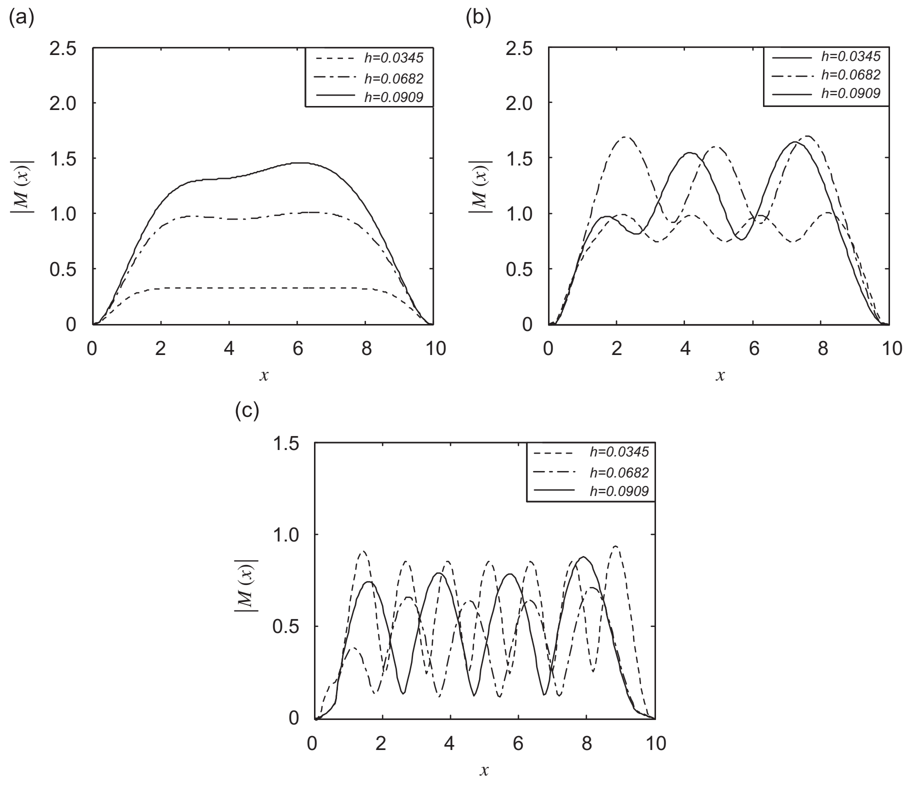

Fig. 6. Amplitude of the bending moment over the plate for different dimensionless thicknesses. (a) $k = 0.8044$, (b) $k = 2.2216$, and (c) $k = 9.0434$.

can be shown that the range of validity of the present method is such that it can should be applicable for predicting the dynamic behaviour of the system under a wider range of parameters, such as are discussed below. Moreover, it may be noted that good agreement between the experimental data and the results obtained by this method is also achieved for long incident waves.

By means of the present method, we calculate the amplitudes of both the plate deflection $|w(x)|$ and the bending moments $|M(x)|$ for different dimensionless thicknesses of the plate $h/a = 0.0345$, 0.0682, 0.0909, as shown in Figs. 5 and 6. The other parameters of the model are the same as above. Fig. 5 show the dimensionless amplitudes of the plate deflection for $k = 0.8044$, 2.2216, 9.0434. Fig. 6 show the dimensionless amplitudes of the plate bending moments for the same wavenumbers.

As can be seen in Figs. 5 and 6, the plate deflection and bending moment vary smoothly over the regions far from the plate edges in the case of the long incident wave ($k = 0.8044$), and the deflection amplitudes reduce while the bending moment amplitudes increase as the plate thickness increases. For $k = 2.2216$, the deflection amplitudes also decrease as the plate thickness increases. However, for $k = 2.2216$, the variations of the deflection amplitude and the bending moment are highly complicated. Moreover, in the vicinity of the plate edge, the variations of the deflection amplitudes and the bending moments are very complex for all of the incident waves considered.

5. Conclusion

In this paper, based on the dynamical theories of water waves and Timoshenko–Mindlin thick plates, the diffraction of surface waves by a floating elastic plate is analysed by using the Wiener–Hopf technique. The calculated results obtained by the present method are in good agreement with the results from the literature (Wu et al., 1995; Tkacheva, 2003). Therefore, the validity of the present method is confirmed, and it can be applied to a wider range of practical cases.

This is relevant because it is known that the results obtained from Mindlin thick plate theory are much closer to physical data than results from the classical theory of thin plates or Euler beams assuming small shear effects. In the context of the development of research in ocean engineering, where composite materials are being widely considered for floating platforms and artificial land, etc., it is necessary to employ the theory of Mindlin thick plates to accurately depict the dynamic behaviour of this kind of very large floating structures, and the effects of the transverse shearing and rotary inertia must be included in the theory.

Acknowledgement

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Appendix A

We will now seek the unknown constants a_1, b_1. From Eqs. (27) and (26b), we obtain

$$X_1(\alpha)K_-(\alpha)\mathrm{e}^{-\mathrm{i}\alpha L} + V_-(\alpha) - U_-(\alpha) = a_1\alpha + b_1. \tag{A.1}$$

Substituting the expressions for $V_-(\alpha)$, $U_-(\alpha)$ into Eq. (A.1), we have

$$X_1(\alpha) = \frac{\mathrm{e}^{\mathrm{i}\alpha L}}{K_-(\alpha)}\left[a_1\alpha + b_1 - \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)}\right]. \tag{A.2}$$

Thus, according to Eqs. (12), (18) and (A.2), we obtain the following expression for the scattered potential by Fourier inversion:

$$\varphi^{(s)}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}\alpha(x-L)}\cosh[\alpha(z+1)]}{K_-(\alpha)K_1(\alpha)\cosh(\alpha)}\left[a_1\alpha + b_1 - \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)}\right]\mathrm{d}\alpha, \tag{A.3}$$

from which we can obtain

$$\frac{\partial\varphi^{(s)}}{\partial z}(x,0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha\mathrm{e}^{-\mathrm{i}\alpha(x-L)}\tanh(\alpha)K_+(\alpha)}{K_2(\alpha)}\left[a_1\alpha + b_1 - \frac{1}{2\pi\mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma}\frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi_-^*(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)}\right]\mathrm{d}\alpha. \tag{A.4}$$

For the outer integral, the contour of the integral must lie completely within the intersection of the domains S_+ and S_-. Hence, the integration contour is chosen on the real axis by passing around the points α_0 and k from below and the points $-\alpha_0$ and $-k$ from above, as shown in Fig. A1.

For the inner integral, the integral function can be defined by means of analytic continuation as a function of α over the entire complex plane. The integration contour is closed in the lower half-plane $\mathrm{Im}\,\alpha < \sigma$ and this integral can be evaluated by using the residue calculus technique. The function $K_+(\zeta)$ has zeros at the points $-\alpha_j$ $(j = -2,-1,0,\ldots)$ and poles at the points $-k$, $-k_j$ $(j = 1,2,3,\ldots)$. In the integral, the pole of the function $\Psi^*_-(\zeta)$ at the point $\zeta = -k$ can be annihilated by the pole of the function $K_+(\zeta)$. Moreover, the points $\zeta = -\alpha_j$ $(j = -2,-1,0,\ldots)$ and $\zeta = \alpha$ are the first-order poles of this integrand. Therefore, we have

$$\frac{1}{2\pi \mathrm{i}}\int_{-\infty+\mathrm{i}\sigma}^{\infty+\mathrm{i}\sigma} \frac{\mathrm{e}^{-\mathrm{i}\zeta L}\Psi^*_-(\zeta)\,\mathrm{d}\zeta}{K_+(\zeta)(\zeta-\alpha)} = -\frac{\mathrm{e}^{-\mathrm{i}\alpha L}\Psi^*_-(\alpha)}{K_+(\alpha)} + \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi^*_-(-\alpha_j)}{K'_+(-\alpha_j)(\alpha_j+\alpha)}, \tag{A.5}$$

where $K'_+(-\alpha_j)$ is the derivative of the function $K_+(-\alpha_j)$ at the points $-\alpha_j$ $(j = -2,-1,0,\ldots)$. Substituting Eq. (A.5) into Eq. (A.4), we have

$$\begin{aligned}\frac{\partial \varphi^{(s)}}{\partial z}(x,0) = {} & \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\alpha \mathrm{e}^{-\mathrm{i}\alpha(x-L)}\tanh(\alpha)K_+(\alpha)}{K_2(\alpha)}(a_1\alpha+b_1)\,\mathrm{d}\alpha + \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha \mathrm{e}^{-\mathrm{i}\alpha x}\tanh(\alpha)\Psi^*_-(\alpha)}{K_2(\alpha)}\,\mathrm{d}\alpha \\ & - \frac{1}{2\pi}\sum_{j=-2}^{\infty}\frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi^*_-(-\alpha_j)}{K'_+(-\alpha_j)}\int_{-\infty}^{\infty}\frac{\alpha \mathrm{e}^{-\mathrm{i}\alpha(x-L)}\tanh(\alpha)K_+(\alpha)}{K_2(\alpha)(\alpha_j+\alpha)}\,\mathrm{d}\alpha.\end{aligned} \tag{A.6}$$

For the second integral of Eq. (A.6), the integration contour is closed by a semi-circle of large radius in the lower-half plane, as shown in Fig. A1, and the following result is obtained by using the residue calculus technique:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha \mathrm{e}^{-\mathrm{i}\alpha x}\tanh(\alpha)\Psi^*_-(\alpha)}{K_2(\alpha)}\,\mathrm{d}\alpha = -\mathrm{e}^{\mathrm{i}kx} - \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m x}\Psi^*_-(-\alpha_m)}{K'_2(-\alpha_m)}. \tag{A.7}$$

For the first and third integral of Eq. (A.6), the integration contour is closed in the upper-half plane, and it passes around the points $-k$, $-\alpha_0$ from above and the points k, α_0 from below along the real axis, as shown in Fig. A1. Moreover, by the residue calculus technique, we can obtain the expression for $\partial\varphi_s/\partial z\ (x,0)$:

$$\begin{aligned}\frac{\partial \varphi^{(s)}}{\partial z}(x,0) = {} & \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)K_+(\alpha_m)}{K'_2(\alpha_m)}(a_1\alpha+b_1)\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)} - \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m x}\Psi^*_-(-\alpha_m)}{K'_2(-\alpha_m)} \\ & - \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)K_+(\alpha_m)\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}}{K'_2(\alpha_m)}\sum_{j=-2}^{\infty}\frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi^*_-(-\alpha_j)}{K'_+(-\alpha_j)(\alpha_j+\alpha_m)} - \mathrm{e}^{\mathrm{i}kx}.\end{aligned} \tag{A.8}$$

From Eqs. (7) and (A.8), we obtain

$$\frac{\partial \varphi}{\partial z}(x,0) = \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)K_+(\alpha_m)}{K'_2(\alpha_m)}(a_1\alpha+b_1)\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)} - \mathrm{i}\sum_{m=-2}^{\infty}\frac{\alpha_m\tanh(\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m x}\Psi^*_-(-\alpha_m)}{K'_2(-\alpha_m)}. \tag{A.9}$$

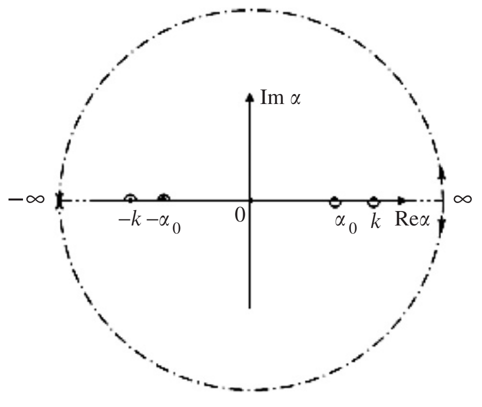

Fig. A1. Schematic of the integration contour for the outer integral in Eq. (A.4).

Taking into account Eqs. (3a), (5c) and (A.9), we obtain

$$w(x) = \mathrm{i}\frac{\partial \varphi}{\partial z}(x,0) = -\sum_{m=-2}^{\infty} \frac{\alpha_m \tanh(\alpha_m) K_+(\alpha_m)}{K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] \mathrm{e}^{-\mathrm{i}\alpha_m(x-L)} + \sum_{m=-2}^{\infty} \frac{\alpha_m \tanh(\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m x}\Psi_-^*(-\alpha_m)}{K_2'(-\alpha_m)}, \tag{A.10}$$

$$F(x) = -\sum_{m=-2}^{\infty} \frac{\alpha_m \tanh(\alpha_m) K_+(\alpha_m)}{K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] \frac{Ca^2\mathrm{e}^{-\mathrm{i}\alpha_m(x-L)}}{D(\gamma^2+\alpha_m^2)} + \sum_{m=-2}^{\infty} \frac{\alpha_m \tanh(\alpha_m)\Psi_-^*(-\alpha_m)}{K_2'(-\alpha_m)} \frac{Ca^2\mathrm{e}^{\mathrm{i}\alpha_m x}}{D(\gamma^2+\alpha_m^2)}. \tag{A.11}$$

According to the boundary conditions (6a) and (6b) at $x = L$, we have

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^3 \tanh(\alpha_m) K_+(\alpha_m) N_1(\alpha_m)}{K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] - \sum_{m=-2}^{\infty} \frac{\alpha_m^3 \tanh(\alpha_m)\Psi_-^*(-\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m L} N_1(\alpha_m)}{K_2'(-\alpha_m)} = 0, \tag{A.12}$$

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^2 \tanh(\alpha_m) K_+(\alpha_m) N_2(\alpha_m)}{K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] + \sum_{m=-2}^{\infty} \frac{\alpha_m^2 \tanh(\alpha_m)\Psi_-^*(-\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m L} N_2(\alpha_m)}{K_2'(-\alpha_m)} = 0, \tag{A.13}$$

where $N_1(\alpha) = 1/(\alpha^2+\gamma^2)$, $N_2(\alpha) = q/(\gamma^2+\alpha^2)-1$, $q = Ca^2/D$.

From the dispersion relations (9) and (10), we have

$$\alpha_m^n \tanh(\alpha_m) = -\frac{\alpha_m^{n-1} b(\alpha_m) K_1(\alpha_m)}{\beta(\alpha_m)} \quad (n = 2, 3). \tag{A.14}$$

Substituting Eq. (A.14) into Eqs. (A.12) and (A.13), we obtain

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^2 b(\alpha_m) K_1(\alpha_m) K_+(\alpha_m) N_1(\alpha_m)}{\beta(\alpha_m) K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] - \sum_{m=-2}^{\infty} \frac{\alpha_m^2 b(\alpha_m) K_1(\alpha_m)\Psi_-^*(-\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m L} N_1(\alpha_m)}{\beta(\alpha_m) K_2'(-\alpha_m)} = 0, \tag{A.15}$$

$$\sum_{m=-2}^{\infty} \frac{\alpha_m b(\alpha_m) K_1(\alpha_m) K_+(\alpha_m) N_2(\alpha_m)}{\beta(\alpha_m) K_2'(\alpha_m)} \left[a_1\alpha_m + b_1 - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_-^*(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)} \right] + \sum_{m=-2}^{\infty} \frac{\alpha_m b(\alpha_m) K_1(\alpha_m)\Psi_-^*(-\alpha_m)\mathrm{e}^{\mathrm{i}\alpha_m L} N_2(\alpha_m)}{\beta(\alpha_m) K_2'(-\alpha_m)} = 0. \tag{A.16}$$

The sums of the infinite series in the above equations are replaced by integrals. The integration contours $C_\pm$ are chosen along the real axis from $-\infty$ to ∞ within the intersection of the domain S_+ and S_-. The subscripts $\pm$ of the contour $C_\pm$ mean that the integration contour lies above and below the origin as shown in Figs. A2 and A3. C_+ is used to indicate that the integration contour passes around the points $-k$, $-\alpha_0$, $\mathrm{i}\gamma$, χ_3, χ_2, χ_1 from above and the points k, α_0 from below along the real axis. Moreover, C_- is used to indicate that the integration contour passes around the points $-k$, $-\alpha_0$ from above and the points k, α_0, $\mathrm{i}\gamma$, χ_1, χ_3, χ_4 from below. The points χ_1, χ_2, χ_3, χ_4 are the positive real root, the positive imaginary root, the negative real root and the negative imaginary root of the equation $\beta(\alpha) = 0$, respectively.

The integration contour C_+ is chosen and closed in the upper-half plane for the first summation over m in Eqs. (A.15) and (A.16). Similarly, the integration contour C_- is chosen and closed in the lower-half plane for the second

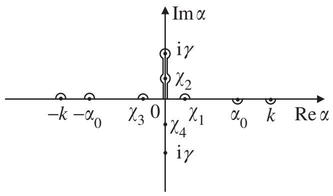

Fig. A2. Schematic of the integration contour C_+.

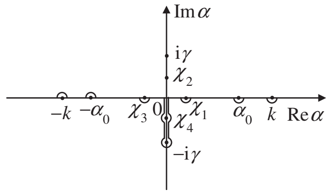

Fig. A3. Schematic of the integration contour C_-.

summation. Therefore, we obtain

$$\frac{1}{2\pi i}\int_{C_+}\frac{\alpha^2 b(\alpha)N_1(\alpha)}{\beta(\alpha)K_-(\alpha)}(a_1\alpha+b_1)\,d\alpha+\frac{1}{2\pi i}\int_{C_-}\frac{e^{-i\alpha L}\alpha^2 b(\alpha)\Psi^*_-(\alpha)N_1(\alpha)\,d\alpha}{\beta(\alpha)K(\alpha)} -\frac{1}{2\pi i}\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi^*_-(-\alpha_j)}{K'_+(-\alpha_j)}\int_{C_+}\frac{\alpha^2 b(\alpha)N_1(\alpha)\,d\alpha}{\beta(\alpha)K_-(\alpha)(\alpha_j+\alpha)}=0, \tag{A.17}$$

$$\frac{1}{2\pi i}\int_{C_+}\frac{\alpha b(\alpha)N_2(\alpha)}{\beta(\alpha)K_-(\alpha)}(a_1\alpha+b_1)\,d\alpha+\frac{1}{2\pi i}\int_{C_-}\frac{\alpha e^{-i\alpha L}b(\alpha)\Psi^*_-(\alpha)N_2(\alpha)\,d\alpha}{\beta(\alpha)K(\alpha)} -\frac{1}{2\pi i}\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi^*_-(-\alpha_j)}{K'_+(-\alpha_j)}\int_{C_+}\frac{\alpha b(\alpha)N_2(\alpha)\,d\alpha}{\beta(\alpha)K_-(\alpha)(\alpha_j+\alpha)}=0. \tag{A.18}$$

Using the residue calculus technique, we can obtain the following system of equations with respect to the unknown constants a_1 and b_1:

$$A_{11}a_1+A_{12}b_1=\sum_{s=1}^{4}\frac{\chi_s^2 b(\chi_s)N_1(\chi_s)}{\beta'(\chi_s)K_-(\chi_s)}\frac{1}{2\pi i}\int_{C_-}\frac{e^{-i\alpha L}\Psi^*_-(\alpha)\,d\alpha}{K_+(\alpha)(-\alpha+\chi_s)} +\frac{i\gamma b(i\gamma)}{2\beta(i\gamma)}\frac{1}{2\pi i}\int_{C_-}\frac{e^{-i\alpha L}\Psi^*_-(\alpha)}{K_+(\alpha)}\left[\frac{1}{K_-(i\gamma)(-\alpha+i\gamma)}+\frac{1}{K_-(-i\gamma)(\alpha+i\gamma)}\right]d\alpha, \tag{A.19}$$

$$A_{21}a_1+A_{22}b_1=\sum_{s=1}^{4}\frac{\chi_s b(\chi_s)N_2(\chi_s)}{\beta'(\chi_s)K_-(\chi_s)}\frac{1}{2\pi i}\int_{C_-}\frac{e^{-i\alpha L}\Psi^*_-(\alpha)\,d\alpha}{K_+(\alpha)(-\alpha+\chi_s)} +\frac{qb(i\gamma)}{2\beta(i\gamma)}\frac{1}{2\pi i}\int_{C_-}\frac{e^{-i\alpha L}\Psi^*_-(\alpha)}{K_+(\alpha)}\left[\frac{1}{K_-(i\gamma)(-\alpha+i\gamma)}-\frac{1}{K_-(-i\gamma)(\alpha+i\gamma)}\right]d\alpha, \tag{A.20}$$

where

$$A_{11}=\frac{1}{2\pi i}\int_{C_+}\frac{\alpha^3 b(\alpha)N_1(\alpha)}{\beta(\alpha)K_-(\alpha)}\,d\alpha,\quad A_{12}=\frac{1}{2\pi i}\int_{C_+}\frac{\alpha^2 b(\alpha)N_1(\alpha)}{\beta(\alpha)K_-(\alpha)}\,d\alpha,$$
$$A_{21}=\frac{1}{2\pi i}\int_{C_+}\frac{\alpha^2 b(\alpha)N_2(\alpha)}{\beta(\alpha)K_-(\alpha)}\,d\alpha,\quad A_{22}=\frac{1}{2\pi i}\int_{C_+}\frac{\alpha b(\alpha)N_2(\alpha)}{\beta(\alpha)K_-(\alpha)}\,d\alpha.$$

We now turn to the other two constants, a_2 and b_2. From Eqs. (28) and (29), we obtain

$$X_1(\alpha)K_+(\alpha)-R_+(\alpha)+S_+(\alpha)+\frac{B(k)}{i(\alpha+k)K_+(k)}=a_2\alpha+b_2. \tag{A.21}$$

Following the same method as used to obtain expression (A.3), we can get the following expression for the diffracted potential $\varphi^{(s)}(x,z)$ by Fourier inversion:

$$\varphi^{(s)}(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\alpha x}\cosh[\alpha(z+1)]}{\cosh(\alpha)K_+(\alpha)K_1(\alpha)}\left[a_2\alpha+b_2-\frac{B(k)}{i(\alpha+k)K_+(k)}+\frac{1}{2\pi i}\int_{-\infty-i\sigma}^{\infty-i\sigma}\frac{e^{i\zeta L}\Psi_+(\zeta)\,d\zeta}{K_-(\zeta)(\zeta-\alpha)}\right]d\alpha. \tag{A.22}$$

After taking the derivative of Eq. (A.22) with respect to $z=0$, we obtain

$$\frac{\partial\varphi^{(s)}}{\partial z}(x,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha e^{-i\alpha x}\tanh(\alpha)K_-(\alpha)}{K_2(\alpha)}\left[a_2\alpha+b_2-\frac{B(k)}{i(\alpha+k)K_+(k)}+\frac{1}{2\pi i}\int_{-\infty-i\sigma}^{\infty-i\sigma}\frac{e^{i\zeta L}\Psi_+(\zeta)\,d\zeta}{K_-(\zeta)(\zeta-\alpha)}\right]d\alpha. \tag{A.23}$$

The function $K_-(\zeta)$ has zeros at the points α_j ($j=-2,-1,0,\ldots$) and first-order poles at the points k, k_j ($j=1,2,3,\ldots$) and $\zeta=\alpha$. For the inner integral, we close the integration contour in the upper half-plane and evaluate this integral with the residue calculus technique. Hence, we obtain

$$\frac{1}{2\pi i}\int_{-\infty-i\sigma}^{\infty-i\sigma}\frac{e^{i\zeta L}\Psi_+(\zeta)\,d\zeta}{K_-(\zeta)(\zeta-\alpha)}=\frac{e^{i\alpha L}\Psi_+(\alpha)}{K_-(\alpha)}+\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)(\alpha_j-\alpha)}.$$

Substituting the above equation into Eq. (A.23), we can get

$$\begin{aligned}\frac{\partial\varphi^{(s)}}{\partial y}(x,0)=&\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha e^{-i\alpha x}\tanh(\alpha)K_-(\alpha)}{K_2(\alpha)}\left[a_2\alpha+b_2-\frac{B(k)}{i(\alpha+k)K_+(k)}\right]d\alpha\\&+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha e^{-i\alpha(x-L)}\tanh(\alpha)\Psi_+(\alpha)}{K_2(\alpha)}\,d\alpha+\frac{1}{2\pi}\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)}\int_{-\infty}^{\infty}\frac{\alpha e^{-i\alpha x}\tanh(\alpha)K_-(\alpha)}{K_2(\alpha)(\alpha_j-\alpha)}\,d\alpha.\end{aligned} \tag{A.24}$$

For the first and third integrals, the integration contours are closed in the lower half-plane and for the second integral in the upper half-plane as shown in Fig. A1. Using the residue calculus technique, we can obtain the following expression for $\partial\varphi^{(s)}/\partial z\ (x,0)$:

$$\begin{aligned}\frac{\partial\varphi^{(s)}}{\partial z}(x,0)=&-i\sum_{m=-2}^{\infty}\frac{e^{i\alpha_m x}\alpha_m\tanh(\alpha_m)K_-(-\alpha_m)}{K'_2(-\alpha_m)}\left[-a_2\alpha_m+b_2-\frac{B(k)}{i(k-\alpha_m)K_+(k)}+\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)(\alpha_j+\alpha_m)}\right]\\&-e^{i\gamma x}+i\sum_{m=-2}^{\infty}\frac{e^{-i\alpha_m(x-L)}\alpha_m\tanh(\alpha_m)\Psi_+(\alpha_m)}{K'_2(\alpha_m)}.\end{aligned} \tag{A.25}$$

From Eqs. (7) and (A.25), we have

$$\begin{aligned}\frac{\partial\varphi}{\partial z}(x,0)=&-i\sum_{m=-2}^{\infty}\frac{e^{i\alpha_m x}\alpha_m\tanh(\alpha_m)K_-(-\alpha_m)}{K'_2(-\alpha_m)}\left[-a_2\alpha_m+b_2-\frac{B(k)}{i(k-\alpha_m)K_+(k)}+\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)(\alpha_j+\alpha_m)}\right]\\&+i\sum_{m=-2}^{\infty}\frac{e^{-i\alpha_m(x-L)}\alpha_m\tanh(\alpha_m)\Psi_+(\alpha_m)}{K'_2(\alpha_m)}.\end{aligned} \tag{A.26}$$

Similarly, taking into account Eqs. (3a), (5c) and (A.26), we obtain

$$\begin{aligned}w(x)=i\frac{\partial\varphi}{\partial z}(x,0)=&\sum_{m=-2}^{\infty}\frac{e^{i\alpha_m x}\alpha_m\tanh(\alpha_m)K_-(-\alpha_m)}{K'_2(-\alpha_m)}\left[-a_2\alpha_m+b_2-\frac{B(k)}{i(k-\alpha_m)K_+(k)}+\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)(\alpha_j+\alpha_m)}\right]\\&-\sum_{m=-2}^{\infty}\frac{e^{-i\alpha_m(x-L)}\alpha_m\tanh(\alpha_m)\Psi_+(\alpha_m)}{K'_2(\alpha_m)},\end{aligned} \tag{A.27}$$

$$\begin{aligned}F(x)=&\sum_{m=-2}^{\infty}\frac{q\alpha_m\tanh(\alpha_m)K_-(-\alpha_m)N_1(\alpha_m)e^{i\alpha_m x}}{K'_2(-\alpha_m)}\left[-a_2\alpha_m+b_2-\frac{B(k)}{i(k-\alpha_m)K_+(k)}+\sum_{j=-2}^{\infty}\frac{e^{i\alpha_j L}\Psi_+(\alpha_j)}{K'_-(\alpha_j)(\alpha_j+\alpha_m)}\right]\\&-\sum_{m=-2}^{\infty}\frac{q\alpha_m\tanh(\alpha_m)\Psi_+(\alpha_m)N_1(\alpha_m)e^{-i\alpha_m(x-L)}}{K'_2(\alpha_m)}.\end{aligned} \tag{A.28}$$

Substituting Eqs. (A.27) and (A.28) into the boundary conditions (6a) and (6b) at $x = 0$, we can obtain the following two equations:

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^3 \tanh(\alpha_m) K_-(-\alpha_m) N_1(\alpha_m)}{K_2'(-\alpha_m)} \left[-a_2\alpha_m + b_2 - \frac{B(k)}{\mathrm{i}(k-\alpha_m)K_+(k)} + \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_+(\alpha_j)}{K_-'(\alpha_j)(\alpha_j+\alpha_m)} \right] - \sum_{m=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_m L}\alpha_m^3 \tanh(\alpha_m)\Psi_+(\alpha_m)N_1(\alpha_m)}{K_2'(\alpha_m)} = 0, \tag{A.29}$$

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^2 \tanh(\alpha_m) K_-(-\alpha_m) N_2(\alpha_m)}{K_2'(-\alpha_m)} \left[-a_2\alpha_m + b_2 - \frac{B(k)}{\mathrm{i}(k-\alpha_m)K_+(k)} + \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\alpha_j L}\Psi_+(\alpha_j)}{K_-'(\alpha_j)(\alpha_j+\alpha_m)} \right] + \sum_{m=-2}^{\infty} \frac{\alpha_m^2\mathrm{e}^{\mathrm{i}\alpha_m L} \tanh(\alpha_m)\Psi_+(\alpha_m)N_2(\alpha_m)}{K_2'(\alpha_m)} = 0. \tag{A.30}$$

Following a similar procedure as above, we find the expressions for the two unknown constants a_2, b_2:

$$a_2 = \frac{P_{22}Q_1 - P_{12}Q_2}{(P_{11}P_{22} - P_{12}P_{21})} + \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[N_2(\chi_s)P_{12} - \chi_s N_1(\chi_s)P_{22}]}{\beta'(\chi_s)K_+(\chi_s)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)\,\mathrm{d}\alpha}{K_-(\alpha)(\alpha-\chi_s)} + \frac{b(\mathrm{i}\gamma)(qP_{12} - \mathrm{i}\gamma P_{22})}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(\mathrm{i}\gamma)(\alpha-\mathrm{i}\gamma)}\,\mathrm{d}\alpha + \frac{b(\mathrm{i}\gamma)(qP_{12} + \mathrm{i}\gamma P_{22})}{2\beta(\mathrm{i}\gamma)(P_{11}P_{22} - P_{12}P_{21})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(-\mathrm{i}\gamma)(\alpha+\mathrm{i}\gamma)}\,\mathrm{d}\alpha, \tag{A.31}$$

$$b_2 = \frac{P_{21}Q_1 - P_{11}Q_2}{(P_{12}P_{21} - P_{11}P_{22})} + \sum_{s=1}^{4} \frac{\chi_s b(\chi_s)[N_2(\chi_s)P_{11} - \chi_s N_1(\chi_s)P_{21}]}{\beta'(\chi_s)K_+(\chi_s)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)\,\mathrm{d}\alpha}{K_-(\alpha)(\alpha-\chi_s)} + \frac{b(\mathrm{i}\gamma)(qP_{11} - \mathrm{i}\gamma P_{21})}{2\beta(\mathrm{i}\gamma)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(\mathrm{i}\gamma)(\alpha-\mathrm{i}\gamma)}\,\mathrm{d}\alpha + \frac{b(\mathrm{i}\gamma)(qP_{11} + \mathrm{i}\gamma P_{21})}{2\beta(\mathrm{i}\gamma)(P_{12}P_{21} - P_{11}P_{22})} \frac{1}{2\pi\mathrm{i}} \int_{C_+} \frac{\mathrm{e}^{\mathrm{i}\alpha L}\Psi_+(\alpha)}{K_-(\alpha)K_+(-\mathrm{i}\gamma)(\alpha+\mathrm{i}\gamma)}\,\mathrm{d}\alpha, \tag{A.32}$$

where

$$P_{11} = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha^3 b(\alpha)N_1(\alpha)\,\mathrm{d}\alpha}{\beta(\alpha)K_+(\alpha)}, \quad P_{12} = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha^2 b(\alpha)N_1(\alpha)\,\mathrm{d}\alpha}{\beta(\alpha)K_+(\alpha)},$$

$$P_{21} = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha^2 b(\alpha)N_2(\alpha)\,\mathrm{d}\alpha}{\beta(\alpha)K_+(\alpha)}, \quad P_{22} = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha b(\alpha)N_2(\alpha)\,\mathrm{d}\alpha}{\beta(\alpha)K_+(\alpha)},$$

$$Q_1 = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha^2 b(\alpha)BN_1(\alpha)\,\mathrm{d}\alpha}{\mathrm{i}(k+\alpha)K_+(k)\beta(\alpha)K_+(\alpha)}, Q_2 = \frac{1}{2\pi\mathrm{i}} \int_{C_-} \frac{\alpha b(\alpha)BN_2(\alpha)\,\mathrm{d}\alpha}{\mathrm{i}(k+\alpha)K_+(k)\beta(\alpha)K_+(\alpha)}.$$

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